Chapter 7 : Factorisation

ANSWER KEYS

EXERCISE 7.1

1. (*i*) $6x + 9$

we can write,

$6x = 2 \times 3 \times x$

$9 = 3 \times 3$

$6x + 9 = 2 \times 3 \times x + 3 \times 3$

$= 3 \times (2x + 3)$

$\therefore \quad 6x + 9 = 3(2x + 3)$

(*ii*) $21x + 14$

we can write,

$21x = 3 \times 7 \times x$

$14 = 2 \times 7$

$21x + 14 = 3 \times 7 \times x + 2 \times 7$

$= 7(3x + 2)$

$\therefore \quad 21x + 14 = 7(3x + 2)$

(*iii*) $12x^2 - 9x$

we can write,

$12x^2 = 2 \times 2 \times 3 \times x \times x$

$9x = 3 \times 3 \times x$

$12x^2 - 9x = 2 \times 2 \times 3 \times x \times x - 3 \times 3 \times x$

$= 3 \times x(2 \times 2 \times x - 3)$

$\therefore \quad 12x^2 - 9x = 3x(4x - 3)$

2. (*i*) $8x^2 - 24xy$

we can write,

$8x^2 = 2 \times 2 \times 2 \times x \times x$

$24xy = 2 \times 2 \times 2 \times 3 \times x \times y$

$8x^2 - 24xy = 2 \times 2 \times 2 \times x(x - 3y)$

$= 8x(x - 3y)$

$\therefore \quad 8x^2 - 24xy = 8x(x - 3y)$

(*ii*) $10xy^2 - 15x^2y$

we can write,

$10xy^2 = 2 \times 5 \times x \times y \times y$

$15x^2y = 3 \times 5 \times x \times x \times y$

$10xy^2 - 15x^2y = 2 \times 5 \times x \times y \times y - 3 \times 5 \times x \times x \times y$

$= 5xy\ (2y - 3x)$

$\therefore \quad 10xy^2 - 15x^2y = 5xy(2y - 3x)$

(*iii*) $27x^3 - 18x^2$

we can write,

$27x^3 = 3 \times 3 \times 3 \times x \times x \times x$

$18x^2 = 2 \times 3 \times 3 \times x \times x$

$27x^3 - 18x^2 = 3 \times 3 \times 3 \times x \times x \times x - 2 \times 3 \times 3 \times x \times x$

$= 3 \times 3 \times x \times x\ (3x - 2)$

$= 9x^2(3x - 2)$

$\therefore \quad 27x^3 - 18x^2 = 9x^2(3x - 2)$

3. (*i*) $6xy - 9yz$

we can write,

$6xy = 2 \times 3 \times x \times y$

$9yz = 3 \times 3 \times y \times z$

$6xy - 9yz = 2 \times 3 \times x \times y - 3 \times 3 \times y \times z$

$= 3y(2x - 3z)$

$\therefore \quad 6xy - 9yz = 3y(2x - 3z)$

(*ii*) $6x + 18$

we can write,

$6x = 2 \times 3 \times x$

$18 = 2 \times 3 \times 3$

$6x + 18 = 2 \times 3 \times x + 2 \times 3 \times 3$

$= 2 \times 3(x + 3)$

$\therefore \quad 6x + 18 = 6(x + 3)$

(*iii*) $10ax - 15ay$

we can write,

$10ax = 2 \times 5 \times a \times x$

$15ay = 3 \times 5 \times a \times y$

$10ax - 15ay = 2 \times 5 \times a \times x - 3 \times 5 \times a \times y$

$= 5 \times a(2x - 3y)$

$\therefore \quad 10ax - 15ay = 5a(2x - 3y)$

4. (*i*) $x^2 + 3x$

we can write,

$x^2 = x \times x$

$3x = 3 \times x$

$x^2 + 3x = x \times x + 3 \times x$

$= x(x + 3)$

$\therefore \quad x^2 + 3x = x(x + 3)$

(*ii*) $21m^2 - 14m$

we can write,

$21m^2 = 3 \times 7 \times m \times m$

$14m = 2 \times 7 \times m$

$21m^2 - 14m = 3 \times 7 \times m \times m - 2 \times 7 \times m$

$= 7 \times m(3m - 2)$

$\therefore \quad 21m^2 - 14m = 7m(3m - 2)$

(*iii*) $9y^2 + 6y$

we can write,

$9y^2 = 3 \times 3 \times y \times y$

$6y = 2 \times 3 \times y$

$\therefore \quad 9y^2 + 6y = 3 \times 3 \times y \times y + 2 \times 3 \times y$

$= 3 \times y \times (3y + 2)$

$9y^2 + 6y = 3y(3y + 2)$

(*iv*) $16x^2 - 24$

we can write,

$16x^2 = 2 \times 2 \times 2 \times 2 \times x \times x$

$24 = 2 \times 2 \times 2 \times 3$

$16x^2 - 24 = 2 \times 2 \times 2 \times 2 \times x \times x - 2 \times 2 \times 2 \times 3$

$= 2 \times 2 \times 2 \times (2x^2 - 3)$

$= 8(2x^2 - 3)$

$\therefore \quad 16x^2 - 24 = 8(2x^2 - 3)$

5. (*i*) $-3x^2 + 3xy - 4xz$

we can write,

$3x^2 = 3 \times x \times x$

$3xy = 3 \times x \times y$

$4xz = 2 \times 2 \times x \times z$

$-3x^2 + 3xy - 4xz = -3 \times x \times x + 3 \times x \times y - 2 \times 2 \times x \times z$

$= -x(3x - 3y + 4z)$

$-3x^2 + 3xy - 4xz = -x(3x - 3y + 4z)$

(*ii*) $x^2yz + xy^2z + xyz^2$

we can write,

$x^2yz = x \times x \times y \times z$

$xy^2z = x \times y \times y \times z$

$xyz^2 = x \times y \times z \times z$

$\therefore x^2yz + xy^2z + xyz^2 = (x \times x \times y \times z + x \times y \times y \times z + x \times y \times z \times z)$

$= xyz(x + y + z)$

$x^2yz + xy^2z + xyz^2 = xyz(x + y + z)$

(*iii*) $x^2y - xy^2 + x^2y^2$

we can write,

$x^2y = x \times x \times y$

$xy^2 = x \times y \times y$

$x^2y^2 = x \times x \times y \times y$

$x^2y - xy^2 + x^2y^2 = x \times x \times y - x \times y \times y + x \times x \times y \times y$

$= xy(x - y + xy)$

$\therefore \quad x^2y - xy^2 + x^2y^2 = xy(x - y + xy)$

6. $14a^2 - 28a^2b^2 + 21a^4$

we can write,

$14a^2 = 2 \times 7 \times a \times a$

$28a^2b^2 = 2 \times 2 \times 7 \times a \times a \times b \times b$

$21a^4 = 3 \times 7 \times a \times a \times a \times a$

$14a^2 - 28a^2b^2 + 21a^4 = 2 \times 7 \times a \times a - 2 \times 2 \times 7 \times a \times a \times b \times b + 3 \times 7 \times a \times a \times a \times a$

$= 7 \times a \times a(2 - 4 \times b^2 + 3 \times a^2)$

$= 7a^2(2 - 4b^2 + 3a^2)$

$\therefore \; 14a^2 - 28a^2b^2 + 21a^4 = 7a^2(2 - 4b^2 + 3a^2)$

7. $a^4b - 6a^2b^2 + 3ab^3$

we can write,

$a^4b = a \times a \times a \times a \times b$

$6a^2b^2 = 2 \times 3 \times a \times a \times b \times b$

$3ab^3 = 3 \times a \times b \times b \times b$

$a^4b - 6a^2b^2 + 3ab^3 = a \times a \times a \times a \times b - 2 \times 3 \times a \times a \times b \times b + 3 \times a \times b \times b \times b$

$= a \times b(a^3 - 6ab + 3b^2)$

$a^4b - 6a^2b^2 + 3ab^3 = ab(a^3 - 6ab + 3b^2)$

8. $x^4y^4 + x^4y^2 - x^2y^4$

$= x^2y^2(x^2y^2 + x^2 - y^2)$

9. $9x^3y^2 + 18xy^4 - 27x^2y^3$

$= 9xy^2(x^2 + 2y^2 - 3xy)$

10. $-5x^2 + 5xy - 5xz$

$= -5x(x - y + z)$

11. $3x^2y^2 - 2xy^2 + 6x^2y$

$= xy(3xy - 2y + 6x)$

12. $12a^3b^2 + 18ab^4 - 6a^2b^3$

$= 6ab^2(2a^2 + 3b^2 - ab)$

13. $16x^2y + 12bxy = 4xy(4x + 3b)$

14. $3x^3 - 6x^2 + 12x = 3x(x^2 - 2x + 4)$

15. $64x^6y^6 - 72x^4y^6 = 8x^4y^6(8x^2 - 9)$

16. $6x^3y^2 + 8x^2y - 2xy = 2xy(3x^2y + 4x - 1)$

17. $15a^3 + 45a^2 + 30a = 15a(a^2 + 3a + 2)$

18. $18a^2bc - 12ab^2c + 6abc^2$

$= 6abc(3a - 2b + c)$

19. $8(x - 3y)^2 + 4(x - 3y)$

$= 4(x - 3y)\ [2(x - 3y) + 1]$

[Taking $4(x - 3y)$ common]

$= 4(x - 3y)\ (2x - 6y + 1)$

20. $3x(a - 3b) - y(a - 3b)$

$= (a - 3b)\ (3x - y)$

[Taking $(a - 3b)$ common]

21. $ab^2 + ab + b + 1$

$= (ab^2 + ab) + (b + 1)$

$= ab(b + 1) + (b + 1)$

$= (b + 1)\ (ab + 1)$

[Taking $(b + 1)$ common]

22. $x^3 + y^3 + xy\ (x + y)$

$= x^3 + y^3 + x^2y + xy^2$

$= (x^3 + x^2y) + (y^3 + xy^2)$

$= x^2(x + y) + y^2(y + x)$

$= x^2(x + y) + y^2(x + y)$

$= (x + y)\ (x^2 + y^2)$

[Taking $(x + y)$ common]

23. $3xy - y^2 + 6xz - 2yz$

$= (3xy - y^2) + (6xz - 2yz)$

$= y(3x - y) + 2z(3x - y)$

$= (3x - y)\ (y + 2z)$

[Taking $(3x - y)$ common]

24. $x^2y + xy + x + 1$

$= (x^2y + xy) + (x + 1)$

$= xy(x + 1) + (x + 1)$

$= (x + 1)\ (xy + 1)$

[Taking $(x + 1)$ common]

25. $ab(x^2 + y^2) + xy\ (a^2 + b^2)$

$= abx^2 + aby^2 + xya^2 + xyb^2$

$= (abx^2 + xya^2) + (aby^2 + xyb^2)$

$= ax(bx + ay) + by(ay + bx)$

$= ax(ay + bx) + by(ay + bx)$

$= (ay + bx)\ (ax + by)$

26. $a^2 - a(x + 3b) + 3bx$

$= a^2 - ax - 3ab + 3bx$

$= (a^2 - ax) - (3ab - 3bx)$

$= a(a - x) - 3b(a - x)$

$= (a - x)\ (a - 3b)$

[Taking $(a - x)$ common]

27. $(a - b) + (a - b)^2$

$= (a - b)\ [1 + (a - b)]$

$= (a - b)\ (1 + a - b)$

28. $(x + y)\ (3a - b) - (2x - 3y)\ (3a - b)$

$= (3a - b)\ [(x + y) - (2x - 3y)]$

$= (3a - b)\ [x + y - 2x + 3y]$

$= (3a - b)\ (- x + 4y)$

$= (3a - b)\ (4y - x)$

29. $7(a + 2b)^2 - 21(a + 2b)$

$= 7(a + 2b)\ [(a + 2b) - 3]$

[Taking $7(a + 2b)$ common]

$= 7(a + 2b)\ (a + 2b - 3)$

30. $4a(5a - 6b) - 12a^2(5a - 6b)$

$= 4a(5a - 6b)\ (1 - 3a)$

EXERCISE 7.2

1. $9a^2 - 16b^2 = (3a)^2 - (4b)^2$

$= (3a + 4b)\ (3a - 4b)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

2. $16x^4 - 144x^2 = 16x^2(x^2 - 9)$

$= 16x^2\ (x^2 - 3^2)$ [Taking $16x^2$ common]

$= 16x^2\ (x + 3)\ (x - 3)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

3. $3x^2 - 12 = 3(x^2 - 4)$

$= 3(x^2 - 2^2)$

$= 3(x + 2)\ (x - 2)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

4. $x^2 - 64y^2 = x^2 - (8y)^2$

$= (x + 8y)\ (x - 8y)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

5. $(a + b)^2 - (a - b)^2$

$= [(a + b) + (a - b)]\ [(a + b) - (a - b)]$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

$= (a + b + a - b)\ (a + b - a + b)$

$= (2a)\ (2b) = 4ab$

6. $16a^2 - 225y^2 = (4a)^2 - (15y)^2$

$= (4a + 15y)\ (4a - 15y)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

7. $(2x + 3y)^2 - 9z^2 = (2x + 3y)^2 - (3z)^2$

$= (2x + 3y + 3z)\ (2x + 3y - 3z)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

8. $9x^2 - \dfrac{1}{144} = (3x)^2 - \left(\dfrac{1}{12}\right)^2$

$= \left(3x + \dfrac{1}{12}\right)\left(3x - \dfrac{1}{12}\right)$

9. $3x^5 - 243x = 3x(x^4 - 81)$

$= 3x[(x^2)^2 - (9)^2]$

$= 3x(x^2 + 9)\ (x^2 - 9)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

$= 3x(x^2 + 9)\ (x^2 - 3^2)$

$= 3x(x^2 + 9)(x + 3)(x - 3)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

10. $16(x + y)^2 - 25(x - 3y)^2$

$= [4(x + y)]^2 - [5(x - 3y)]^2$

$= [4(x + y) + 5(x - 3y)]$

$[4(x + y) - 5(x - 3y)]$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

$= (4x + 4y + 5x - 15y)$

$(4x + 4y - 5x + 15y)$

$= (9x - 11y)\ (-x + 19y)$

$= -\ (9x - 11y)\ (x - 19y)$

11. $81 - p^4 = [9^2 - (p^2)^2]$

$= (9 + p^2)\ (9 - p^2)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

$= (9 + p^2)\ (3^2 - p^2)$

$= (9 + p^2)\ (3 + p)\ (3 - p)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

12. $3x^4 - 3 = 3(x^4 - 1)$ [Taking 3 common]

$= 3[(x^2)^2 - (1)^2]$

$= 3(x^2 + 1)\ (x^2 - 1)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

$= 3(x^2 + 1)\ (x + 1)\ (x - 1)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

13. $(2a + 3b)^2 - 25c^2$

$= (2a + 3b)^2 - (5c)^2$

$= (2a + 3b + 5c)\ (2a + 3b - 5c)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

14. $3 - 75x^2 = 3(1 - 25x^2)$

$= 3[1 - (5x)^2] = 3(1 + 5x)\ (1 - 5x)$

15. $x^4 - 16y^4 = [(x^2)^2 - (4y^2)^2]$

$= (x^2 + 4y^2)\ (x^2 - 4y^2)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

$= (x^2 + 4y^2)\ [x^2 - (2y)^2]$

$= (x^2 + 4y^2)\ (x + 2y)\ (x - 2y)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

16. $x^{12}y^4 - x^4y^{12} = x^4y^4(x^8 - y^8)$

$= x^4y^4[(x^4)^2 - (y^4)^2]$

$= x^4y^4(x^4 + y^4)\ (x^4 - y^4)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

$= x^4y^4\ (x^4 + y^4)\ [(x^2)^2 - (y^2)^2]$

$= x^4y^4(x^4 + y^4)\ (x^2 + y^2)\ (x^2 - y^2)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

$= x^4y^4(x^4 + y^4)\ (x^2 + y^2)\ (x + y)\ (x - y)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

17. $16a^2 - \dfrac{25}{9a^2} = (4a)^2 - \left(\dfrac{5}{3a}\right)^2$

$= \left(4a + \dfrac{5}{3a}\right)\left(4a - \dfrac{5}{3a}\right)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

18. $(x + 2)^2 - (x + 4)^2$

$= [(x + 2) + (x + 4)]\ [(x + 2) - (x + 4)]$

$= (x + 2 + x + 4)\ (x + 2 - x - 4)$

$= (2x + 6)(- 2)$

$= - 2 \times 2\ (x + 3)$

$= -4(x + 3)$

19. $(2 + 3a)^2 - 36a^2 = (2 + 3a)^2 - (6a)^2$

$= (2 + 3a + 6a)\ (2 + 3a - 6a)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

$= (2 + 9a)\ (2 - 3a)$

20. $x^3 - 225x = x(x^2 - 225)$ [Taking x common]

$= x[x^2 - (15)^2]$

$= x(x + 15)\ (x - 15)$

21. $(3x - 4y)^4 - x^4$

$= [(3x - 4y)^2]^2 - (x^2)^2$

$= [(3x - 4y)^2 + x^2]\ [(3x - 4y)^2 - x^2]$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

$= [(3x - 4y)^2 + x^2]\ (3x - 4y + x)\ (3x - 4y - x)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

$= [9x^2 + 16y^2 - 24xy + x^2]\ (4x - 4y)\ (2x - 4y)$

$\left\{\begin{array}{l}\because \quad a^2 - b^2 = (a + b)(a - b) \\ \text{and} \quad\ (a - b)^2 = (a^2 - 2ab + b^2)\end{array}\right\}$

$= (10x^2 + 16y^2 - 24xy)\ (4x - 4y)\ (2x - 4y)$

$= 2 \times 4 \times 2(5x^2 + 8y^2 - 12xy)\ (x - y)\ (x - 2y)$

$= 16(5x^2 + 8y^2 - 12xy)\ (x - y)\ (x - 2y)$

22. $(a - b)^2 - (x - y)^2$

$= [(a - b) + (x - y)]\ [(a - b) - (x - y)]$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

$= (a - b + x - y)\ (a - b - x + y)$

23. $x^8 - 1 = (x^4)^2 - (1)^2$

$= (x^4 + 1)\ (x^4 - 1)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

$= (x^4 + 1)\ [(x^2)^2 - (1)^2]$

$= (x^4 + 1)\ (x^2 + 1)\ (x^2 - 1)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

$= (x^4 + 1)\ (x^2 + 1)\ (x + 1)\ (x - 1)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

24. $169a^2 - 144b^2 = (13a)^2 - (12b)^2$

$= (13a + 12b)\ (13a - 12b)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

25. $x^5 - 81x^3 = x^3(x^2 - 81)$

$= x^3(x^2 - 9^2)$

$= x^3(x + 9)(x - 9)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

26. $\dfrac{1}{9}x^2y^2 - \dfrac{1}{16}y^2z^2 = y^2\left[\dfrac{1}{9}x^2 - \dfrac{1}{16}z^2\right]$

$= y^2\left[\left(\dfrac{1}{3}x\right)^2 - \left(\dfrac{1}{4}z\right)^2\right]$

$= y^2\left[\left(\dfrac{1}{3}x + \dfrac{1}{4}z\right)\left(\dfrac{1}{3}x - \dfrac{1}{4}z\right)\right]$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

$= y^2\left(\dfrac{x}{3} + \dfrac{z}{4}\right)\left(\dfrac{x}{3} - \dfrac{z}{4}\right)$

27. $3a^5 - 48a^3 = 3a^3(a^2 - 16)$

$= 3a^3(a^2 - 4^2)$

$= 3a^3\ (a + 4)\ (a - 4)$

$[\because \quad a^2 - b^2 = (a + b)(a - b)]$

28. $(2x - y)^2 - 9z^2$

$= (2x - y)^2 - (3z)^2$

$= [(2x - y) + 3z]\ [(2x - y) - 3z]$

$= (2x - y + 3z)\ (2x - y - 3z)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

29. $125a^2 - 45b^2 = 5(25a^2 - 9b^2)$ [Taking 5 common]

$= 5[(5a)^2 - (3b)^2]$

$= 5(5a + 3b)\ (5a - 3b)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

30. $3x^3y - 243\ xy^3 = 3xy(x^2 - 81y^2)$ [Taking $3xy$ common]

$= 3xy[x^2 - (9y)^2]$

$= 3xy(x + 9y)(x - 9y)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

31. $4 - 16(a-b)^2 = 4[1 - 4(a-b)^2]$

$= 4[(1)^2 - \{2(a-b)\}^2]$

$= 4[1 + 2(a-b)]\,[1 - 2(a-b)]$

$[\because \quad a^2 - b^2 = (a+b)\,(a-b)]$

$= 4(1 + 2a - 2b)\,(1 - 2a + 2b)$

32. $9a^2 - 4b^2 - 3a - 2b$

$= [(3a)^2 - (2b)^2] - (3a + 2b)$

$= (3a + 2b)\,(3a - 2b) - (3a + 2b)$

$[\because \quad a^2 - b^2 = (a+b)\,(a-b)]$

$= (3a + 2b)\,(3a - 2b - 1)$

[Taking $(3a + 2b)$ common]

33. $x^2 - y^2 - x - y = (x^2 - y^2) - (x + y)$

$= (x + y)\,(x - y) - (x + y)$

$= (x + y)\,(x - y - 1)$

34. $x^2 - 3x + 3y - y^2$

$= (x^2 - y^2) - 3(x - y)$

$= (x + y)\,(x - y) - 3(x - y)$

$[\because \quad a^2 - b^2 = (a+b)\,(a-b)]$

$= (x - y)\,(x + y - 3)$

35. $x^7y^3 - x^3y^7 = x^3y^3\,(x^4 - y^4)$

$= x^3y^3[(x^2)^2 - (y^2)^2]$

$= x^3y^3(x^2 + y^2)\,(x^2 - y^2)$

$= x^3y^3(x^2 + y^2)\,(x + y)\,(x - y)$

$[\because \quad a^2 - b^2 = (a+b)\,(a-b)]$

36. $a(a + c) - b(b + c) = a^2 + ac - b^2 - bc$

$= (a^2 - b^2) + ac - bc$

$= (a^2 - b^2) + c(a - b)$

$= (a + b)\,(a - b) + c(a - b)$

$= (a - b)\,[(a + b) + c]$

[Taking $(a - b)$ common]

$= (a - b)\,(a + b + c)$

37. (*i*) $(205)^2 - (195)^2 = (205 + 195)\,(205 - 195)$

$[\because \quad a^2 - b^2 = (a+b)\,(a-b)]$

$= 400 \times 10$

$= 4000$

(*ii*) $(7.6)^2 - (2.4)^2 = (7.6 + 2.4)(7.6 - 2.4)$

$[\because \quad a^2 - b^2 = (a+b)\,(a-b)]$

$= 10 \times 5.2$

$= 52$

(*iii*) $(501)^2 - (499)^2 = (501 + 499)\,(501 - 499)$

$= [\because\ a^2 - b^2 = (a+b)\,(a-b)]$

$= 1000 \times 2$

$= 2000$

EXERCISE 7.3

1. $4x^2 - 12x + 9 = (2x)^2 - 2(2x)(3) + (3)^2$

$= (2x - 3)^2$

$[\because \quad (a-b)^2 = a^2 - 2ab + b^2]$

2. $a^2 - 8a + 16 = a^2 - 2(a)(4) + (4)^2$

$= (a - 4)^2$

$[\because \quad (a-b)^2 = a^2 - 2ab + b^2]$

3. $x^2 - 14x + 49 = x^2 - 2x(7) + (7)^2$

$= (x - 7)^2$

$[\because \quad (a-b)^2 = a^2 - 2ab + b^2]$

4. $25a^2 + 30a + 9 = (5a)^2 + 2(5a)\,(3) + 3^2$

$= (5a + 3)^2$

$[\because \quad (a+b)^2 = a^2 + 2ab + b^2]$

5. $49m^2 + 84mn + 36n^2$

$= (7m)^2 + 2(7m)(6n) + (6n)^2$

$= (7m + 6n)^2$

$[\because \quad (a+b)^2 = a^2 + 2ab + b^2]$

6. $16a^2 - 24a + 9 = (4a)^2 - 2(4a) \times (3) + 3^2$

$= (4a - 3)^2$ $[\because \quad (a-b)^2 = a^2 - 2ab + b^2]$

7. $4x^2 + 4x + 1 = (2x)^2 + 2(2x) \times 1 + (1)^2$

$= (2x + 1)^2$

$[\because \quad (a+b)^2 = a^2 + 2ab + b^2]$

8. $9x^2 + 30xy + 25y^2 = (3x)^2 + 2(3x)(5y) + (5y)^2$

$= (3x + 5y^2)$

$[\because \quad (a+b)^2 = a^2 + 2ab + b^2]$

9. $m^2 - 8mn + 16n^2 = m^2 - 2m(4n) + (4n)^2$

$= (m - 4n)^2$

$[\because \quad (a-b)^2 = a^2 - 2ab + b^2]$

10. $4x^2 - 12xy + 9y^2 = (2x)^2 - 2(2x)(3y) + (3y)^2$

$= (2x - 3y)^2$

$[\because \quad (a-b)^2 = a^2 - 2ab + b^2]$

11. $4x^2 + 20xy + 25y^2 = (2x)^2 + 2(2x)(5y) + (5y)^2$

$= (2x + 5y)^2$

$[\because \quad (a+b)^2 = a^2 + 2ab + b^2]$

12. $9x^2 + 42xy + 49y^2 = (3x)^2 + 2(3x)(7y) + (7y)^2$

$= (3x + 7y)^2$

$[\because \quad (a+b)^2 = a^2 + 2ab + b^2]$

13. $4x^2 - \frac{4}{3}x + \frac{1}{9} = (2x)^2 - 2 \times (2x) \times \frac{1}{3} + \left(\frac{1}{3}\right)^2$

$= \left(2x - \frac{1}{3}\right)^2$

$[\because \quad (a-b)^2 = a^2 - 2ab + b^2]$

14. $a^2 - 2ab + b^2 - c^2 = (a^2 - 2ab + b^2) - c^2$

$= (a - b)^2 - c^2$

$= (a - b + c)\ (a - b - c)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

15. $49 - x^2 - y^2 + 2xy = 49 - (x^2 + y^2 - 2xy)$

$= 49 - (x - y)^2$

$= 7^2 - (x - y)^2$

$[\because \quad (a - b)^2 = a^2 - 2ab + b^2]$

$= [7 + (x - y)]\ [7 - (x - y)]$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

$= (x - y + 7)\ (- x + y + 7)$

16. $x^2 + 6x + 9 - 9y^2 = x^2 + 2x \cdot (3) + (3)^2 - 9y^2$

$= (x + 3)^2 - (3y)^2$

$[\because \quad (a + b)^2 = a^2 + 2ab + b^2]$

$= (x + 3 + 3y)\ (x + 3 - 3y)$

$[\because \quad a^2 - b^2 = (a + b)\ (a - b)]$

$= (x + 3y + 3)\ (x - 3y + 3)$

17. $9x^2 - 3x + \frac{1}{4} = (3x)^2 - 2 \times \frac{1}{2}(3x) + \left(\frac{1}{2}\right)^2$

$= \left(3x - \frac{1}{2}\right)^2$

$[\because \quad (a - b)^2 = a^2 - 2ab + b^2]$

18. $3a^2 + 12a + 12 = 3(a^2 + 4a + 4)$

$= 3[(a^2 + 2 \times a \times 2 + (2)^2]$

$= 3(a + 2)^2$

$[\because \quad (a + b)^2 = a^2 + 2ab + b^2]$

19. $25a^2 - 30a + 9 = (5a)^2 - 2(5a)(3) + 3^2$

$= (5a - 3)^2$

$[\because \quad (a - b)^2 = a^2 - 2ab + b^2]$

20. $9x^2 + 2 + \frac{1}{9x^2} = (3x)^2 + 2(3x)\left(\frac{1}{3x}\right) + \left(\frac{1}{3x}\right)^2$

$= \left(3x + \frac{1}{3x}\right)^2$

$[\because \quad (a + b)^2 = a^2 + 2ab + b^2]$

21. $16p^2 - 24pq + 9q^2 = (4p)^2 - 2(4p)(3q) + (3q)^2$

$= (4p - 3q)^2$

$[\because \quad (a - b)^2 = a^2 - 2ab + b^2]$

22. $3x^2 + 6 + \frac{3}{x^2} = 3\left(x^2 + 2 + \frac{1}{x^2}\right)$

$= 3\left[x^2 + 2x\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2\right]$

$= 3\left(x + \frac{1}{x}\right)^2$

$[\because \quad (a + b)^2 = a^2 + 2ab + b^2]$

23. $4 + 28x + 49x^2 = (2)^2 + 2(2)(7x) + (7x)^2$

$= (2 + 7x)^2$

$[\because \quad (a + b)^2 = a^2 + 2ab + b^2]$

24. $4y^2 - 2y + \frac{1}{4} = (2y)^2 - 2 \times (2y) \times \frac{1}{2} + \left(\frac{1}{2}\right)^2$

$= \left(2y - \frac{1}{2}\right)^2$

$[\because \quad (a - b)^2 = a^2 - 2ab + b^2]$

25. $9x^2 - 30x + 25 - 4y^2$

$= (3x)^2 - 2(3x)(5) + (5)^2 - 4y^2$

$= (3x - 5)^2 - (2y)^2$

$[\because \quad (a - b)^2 = a^2 - 2ab + b^2]$

$= [(3x - 5) + 2y]\ [(3x - 5) - 2y]$

$= (3x + 2y - 5)(3x - 2y - 5)$

$[\because \quad a^2 - b^2 = (a + b)(a - b)]$

26. $a^2 - 9b^2 - 12b - 4 = a^2 - (9b^2 + 12b + 4)$

$= a^2 - [(3b)^2 + 2(3b)(2) + (2)^2]$

$= a^2 - (3b + 2)^2$

$[\because \quad (a + b)^2 = a^2 + 2ab + b^2]$

$= [a + (3b + 2)]\ [a - (3b + 2)]$

$[a^2 - b^2 = (a + b)(a - b)]$

$= (a + 3b + 2)\ (a - 3b - 2)$

27. $\frac{1}{4} - \frac{1}{4}x^2 - 2xy - 4y^2$

$= \frac{1}{4} - \left(\frac{1}{4}x^2 + 2xy + 4y^2\right)$

$= \frac{1}{4} - \left[\left(\frac{x}{2}\right)^2 + 2\left(\frac{x}{2}\right)(2y) + (2y)^2\right]$

$= \frac{1}{4} - \left(\frac{x}{2} + 2y\right)^2 \quad [\because \quad (a + b)^2 = a^2 + 2ab + b^2]$

$= \left(\frac{1}{2}\right)^2 - \left(\frac{x}{2} + 2y\right)^2$

$= \left[\frac{1}{2} + \left(\frac{x}{2} + 2y\right)\right]\left[\frac{1}{2} - \left(\frac{x}{2} + 2y\right)\right]$

$[\because \ a^2 - b^2 = (a + b)(a - b)]$

$= \left(\frac{x}{2} + 2y + \frac{1}{2}\right)\left(-\frac{x}{2} - 2y + \frac{1}{2}\right)$

28. $9 - 12x + 4x^2 - 25y^2 - z^2 - 10yz$

$= (9 - 12x + 4x^2) - (25y^2 + z^2 + 10yz)$

$= [3^2 - 2(3)(2x) + (2x)^2] - [(5y)^2 + 2(5y)\ z + z^2]$

$= (3 - 2x)^2 - (5y + z)^2$

$= [(3 - 2x) + (5y + z)]\ [(3 - 2x) - (5y + z)]$

$= (3 - 2x + 5y + z)\ (3 - 2x - 5y - z)$

EXERCISE 7.4

1. The given expression is $a^2 - 10a + 21$.

The two numbers whose sum is 10 and product is 21 are 7 and 3.

$\therefore \quad a^2 - 10a + 21 = a^2 - (7 + 3)\, a + 21$

$= a^2 - 7a - 3a + 21$

$= a(a - 7) - 3(a - 7)$

$= (a - 7)\,(a - 3)$

2. $x^2 + 5x + 6 = x^2 + (2 + 3)x + 6$

$= x^2 + 2x + 3x + 6$

$= x(x + 2) + 3(x + 2)$

$= (x + 2)(x + 3)$

3. $x^2 + 6x - 16 = x^2 + (8 - 2)x - 16$

$= x^2 + 8x - 2x - 16$

$= x(x + 8) - 2(x + 8)$

$= (x + 8)\,(x - 2)$

4. $x^2 + 7x + 12 = x^2 + (4 + 3)x + 12$

$= x^2 + 4x + 3x + 12$

$= x(x + 4) + 3(x + 4)$

$= (x + 4)\,(x + 3)$

5. $x^2 - 4x - 12 = x^2 - (6 - 2)x - 12$

$= x^2 - 6x + 2x - 12$

$= x(x - 6) + 2(x - 6)$

$= (x - 6)\,(x + 2)$

6. $a^2 + 19a + 78 = a^2 + (13 + 6)\, a + 78$

$= a^2 + 13a + 6a + 78$

$= a(a + 13) + 6(a + 13)$

$= (a + 13)(a + 6)$

7. $x^2 - 2x - 15 = x^2 - (5 - 3)\, x - 15$

$= x^2 - 5x + 3x - 15$

$= x(x - 5) + 3(x - 5)$

$= (x - 5)\,(x + 3)$

8. $x^2 + 9x + 20 = x^2 + (5 + 4)x + 20$

$= x^2 + 5x + 4x + 20$

$= x(x + 5) + 4(x + 5)$

$= (x + 5)(x + 4)$

9. $x^2 + 13x + 40 = x^2 + (8 + 5)x + 40$

$= x^2 + 8x + 5x + 40$

$= x(x + 8) + 5(x + 8)$

$= (x + 8)\,(x + 5)$

10. $x^2 - 12x - 45 = x^2 - (15 - 3)x - 45$

$= x^2 - 15x + 3x - 45$

$= x(x - 15) + 3(x - 15)$

$= (x - 15)\,(x + 3)$

11. $x^2 + x - 72 = x^2 + (9 - 8)\, x - 72$

$= x^2 + 9x - 8x - 72$

$= x(x + 9) - 8\,(x + 9)$

$= (x + 9)\,(x - 8)$

12. $x^2 + 11x - 60 = x^2 + (15 - 4)x - 60$

$= x^2 + 15x - 4x - 60$

$= x(x + 15) - 4(x + 15)$

$= (x + 15)\,(x - 4)$

13. $x^2 + 30x + 81 = x^2 + (27 + 3)x + 81$

$= x^2 + 27x + 3x + 81$

$= x(x + 27) + 3(x + 27)$

$= (x + 27)(x + 3)$

14. $x^2 + 2x - 63 = x^2 + (9 - 7)x - 63$

$= x^2 + 9x - 7x - 63$

$= x(x + 9) - 7(x + 9)$

$= (x + 9)(x - 7)$

15. $x^2 + 11x + 28 = x^2 + (7 + 4)x + 28$

$= x^2 + 7x + 4x + 28$

$= x(x + 7) + 4(x + 7)$

$= (x + 7)(x + 4)$

16. $x^2 - 2x - 15 = x^2 - (5 - 3)x - 15$

$= x^2 - 5x + 3x - 15$

$= x(x - 5) + 3(x - 5)$

$= (x - 5)(x + 3)$

17. $x^2 - 4x - 21 = x^2 - (7 - 3)x - 21$

$= x^2 - 7x + 3x - 21$

$= x(x - 7) + 3(x - 7)$

$= (x - 7)(x + 3)$

18. $x^2 + 5x - 150 = x^2 + (15 - 10)x - 150$

$= x^2 + 15x - 10x - 150$

$= x(x + 15) - 10(x + 15)$

$= (x + 15)(x - 10)$

19. $x^2 - 2x - 99 = x^2 - (11 - 9)x - 99$

$= x^2 - 11x + 9x - 99$

$= x(x - 11) + 9(x - 11)$

$= (x - 11)(x + 9)$

20. $a^2 + 22a + 105 = a^2 + (15 + 7)a + 105$

$= a^2 + 15a + 7a + 105$

$= a(a + 15) + 7(a + 15)$

$= (a + 15)(a + 7)$

21. $x^2 - 13x - 114 = x^2 - (19 - 6)x - 114$

$= x^2 - 19x + 6x - 114$

$= x(x - 19) + 6(x - 19)$

$= (x - 19)(x + 6)$

22. $x^2 + 11x - 102 = x^2 + (17 - 6)x - 102$

$= x^2 + 17x - 6x - 102$

$= x(x + 17) - 6(x + 17)$

$= (x + 17)(x - 6)$

23. $a^2 - 23a + 112 = a^2 - (16 + 7)a + 112$

$= a^2 - 16a - 7a + 112$

$= a(a - 16) - 7(a - 16)$

$= (a - 16)(a - 7)$

24. $a^2 - 22a + 117 = a^2 - (13 + 9)a + 117$
$= a^2 - 13a - 9a + 117$
$= a(a - 13) - 9(a - 13)$
$= (a - 13)(a - 9)$

25. $x^2 + 18x + 65 = x^2 + (13 + 5)x + 65$
$= x^2 + 13x + 5x + 65$
$= x(x + 13) + 5(x + 13)$
$= (x + 13)(x + 5)$

26. $x^2 + 24x + 95 = x^2 + (19 + 5)x + 95$
$= x^2 + 19x + 5x + 95$
$= x(x + 19) + 5(x + 19)$
$= (x + 19)(x + 5)$

27. $x^2 + 15x + 44 = x^2 + (11 + 4)x + 44$
$= x^2 + 11x + 4x + 44$
$= x(x + 11) + 4(x + 11)$
$= (x + 11)(x + 4)$

28. $x^2 + 4x - 77 = x^2 + (11 - 7)x - 77$
$= x^2 + 11x - 7x - 77$
$= x(x + 11) - 7(x + 11)$
$= (x + 11)(x - 7)$

EXERCISE 7.5

1. The given expression is $2x^2 - x - 3$.
The two numbers whose sum is -1, product is $[2 \times (-3)] = 6$ are -3 and 2.
$\therefore \quad 2x^2 - x - 3 = 2x^2 - (3 - 2)x - 3$
$= 2x^2 - 3x + 2x - 3$
$= x(2x - 3) + 1(2x - 3)$
$= (2x - 3)(x + 1)$

2. $3x^2 + 5x - 2 = 3x^2 + (6 - 1)x - 2$
$= 3x^2 + 6x - x - 2$
$= 3x(x + 2) - 1(x + 2)$
$= (x + 2)(3x - 1)$

3. $3x^2 - 8x - 3 = 3x^2 - (9 - 1)x - 3$
$= 3x^2 - 9x + x - 3$
$= 3x(x - 3) + 1(x - 3)$
$= (x - 3)(3x + 1)$

4. $-2x^2 + 3x + 9 = -(2x^2 - 3x - 9)$
$= -[2x^2 - (6 - 3)x - 9]$
$= -[2x^2 - 6x + 3x - 9]$
$= -[2x(x - 3) + 3(x + 3)]$
$= -(x - 3)(2x + 3)$
$= (3 - x)(2x + 3)$

5. $7x^2 - 24x + 9 = 7x^2 - (21 + 3)x + 9$
$= 7x^2 - 21x - 3x + 9$
$= 7x(x - 3) - 3(x - 3)$
$= (x - 3)(7x - 3)$

6. $-5x^2 + 38x - 21 = -(5x^2 - 38x + 21)$
$= -[5x^2 - (35 + 3)x + 21]$
$= -[5x^2 - 35x - 3x + 21]$
$= -[5x(x - 7) - 3(x - 7)]$
$= -(x - 7)(5x - 3)$
$= (7 - x)(5x - 3)$

7. $6y^2 - 11y - 2 = 6y^2 - (12 - 1)y - 2$
$= 6y^2 - 12y + y - 2$
$= 6y(y - 2) + 1(y - 2)$
$= (y - 2)(6y + 1)$

8. $3x^2 - 13x + 12 = 3x^2 - (9 + 4)x + 12$
$= 3x^2 - 9x - 4x + 12$
$= 3x(x - 3) - 4(x - 3)$
$= (x - 3)(3x - 4)$

9. $8x^2 + 26x + 15 = 8x^2 + (20 + 6)x + 15$
$= 8x^2 + 20x + 6x + 15$
$= 4x(2x + 5) + 3(2x + 5)$
$= (2x + 5)(4x + 3)$

10. $3x^2 + 22x + 35 = 3x^2 + (15 + 7)x + 35$
$= 3x^2 + 15x + 7x + 35$
$= 3x(x + 5) + 7(x + 5)$
$= (x + 5)(3x + 7)$

11. $12x^2 + xy - 6y^2 = 12x^2 + (9 - 8)xy - 6y^2$
$= 12x^2 + 9xy - 8xy - 6y^2$
$= 3x(4x + 3y) - 2y(4x + 3y)$
$= (4x + 3y)(3x - 2y)$

12. $6x^2 + 5xy - 6y^2 = 6x^2 + (9 - 4)xy - 6y^2$
$= 6x^2 + 9xy - 4xy - 6y^2$
$= 3x(2x + 3y) - 2y(2x + 3y)$
$= (2x + 3y)(3x - 2y)$

13. $4x^2 - 13xy + 3y^2 = 4x^2 - (12 + 1)xy + 3y^2$
$= 4x^2 - 12xy - xy + 3y^2$
$= 4x(x - 3y) - y(x - 3y)$
$= (x - 3y)(4x - y)$

14. $10x^2 + xy - 21y^2 = 10x^2 + (15 - 14)xy - 21y^2$
$= 10x^2 + 15xy - 14xy - 21y^2$
$= 5x(2x + 3y) - 7y(2x + 3y)$
$= (2x + 3y)(5x - 7y)$

15. $6a^2 - 17ab - 3b^2 = 6a^2 - (18 - 1)ab - 3b^2$
$= 6a^2 - 18ab + ab - 3b^2$
$= 6a(a - 3b) + b(a - 3b)$
$= (a - 3b)(6a + b)$

16. $10a^2 + 17ab + 6b^2 = 10a^2 + (12 + 5)ab + 6b^2$
$= 10a^2 + 12ab + 5ab + 6b^2$
$= 2a(5a + 6b) + b(5a + 6b)$
$= (5a + 6b)(2a + b)$

17. $12x^2 - 11xyz - 15y^2z^2 = 12x^2 - (20 - 9)xyz - 15y^2z^2$
$= 12x^2 - 20xyz + 9xyz - 15y^2z^2$
$= 4x(3x - 5yz) + 3yz(3x - 5yz)$
$= (3x - 5yz)(4x + 3yz)$

18. $4x^2 + 2x - 12 = 2(2x^2 + x - 6)$
$= 2[2x^2 + (4 - 3)x - 6]$
$= 2[2x^2 + 4x - 3x - 6]$
$= 2[2x(x + 2) - 3(x + 2)]$
$= 2(x + 2)(2x - 3)$

19. $x^2 - xy - 6y^2 = x^2 - (3 - 2)xy - 6y^2$
$= x^2 - 3xy + 2xy - 6y^2$
$= x(x - 3y) + 2y(x - 3y)$
$= (x - 3y)\ (x + 2y)$

20. $-2x^2 - 7x - 3 = -[2x^2 + 7x + 3]$
$= -[2x^2 + (6 + 1)x + 3]$
$= -[2x^2 + 6x + x + 3]$
$= -[2x(x + 3) + 1(x + 3)]$
$= -(x + 3)\ (2x + 1)$

21. $6x^2 - 5x - 6 = 6x^2 - (9 - 4)x - 6$
$= 6x^2 - 9x + 4x - 6$
$= 3x(2x - 3) + 2(2x - 3)$
$= (2x - 3)\ (3x + 2)$

22. $2x^2 - 5x + 3 = 2x^2 - (2 + 3)x + 3$
$= 2x^2 - 2x - 3x + 3$
$= 2x(x - 1) - 3(x - 1)$
$= (x - 1)\ (2x - 3)$

23. $10x^2 - 23xy + 12y^2 = 10x^2 - (15 + 8)xy + 12y^2$
$= 10x^2 - 15xy - 8xy + 12y^2$
$= 5x(2x - 3y) - 4y(2x - 3y)$
$= (2x - 3y)\ (5x - 4y)$

24. $2y^2 + y - 3 = 2y^2 + (3 - 2)y - 3$
$= 2y^2 + 3y - 2y - 3$
$= y(2y + 3) - 1(2y + 3)$
$= (2y + 3)\ (y - 1)$

25. $-2x^2 + 7x - 6 = -[2x^2 - 7x + 6]$
$= -[2x^2 - (4 + 3)x + 6]$
$= -[2x^2 - 4x - 3x + 6]$
$= -[2x(x - 2) - 3(x - 2)]$
$= -(x - 2)\ (2x - 3)$
$= (2 - x)\ (2x - 3)$

EXERCISE 7.6

1. $\dfrac{36x^3y^2}{4xy} = \dfrac{36}{4} \times \dfrac{\cancel{x} \times x \times x \times y \times \cancel{y}}{\cancel{x} \times \cancel{y}}$
$= 9 \times x \times x \times y$
$= 9x^2y$

2. $\dfrac{24xy^2z^2}{-6xy^2z} = \dfrac{24}{-6} \times \dfrac{\cancel{x} \times \cancel{y} \times \cancel{y} \times z \times \cancel{z}}{\cancel{x} \times \cancel{y} \times \cancel{y} \times \cancel{z}}$
$= -4z$

3. $\dfrac{-81x^3y^2z}{-9x^2y} = \dfrac{-81}{-9} \times \dfrac{\cancel{x} \times \cancel{x} \times x \times \cancel{y} \times y \times z}{\cancel{x} \times \cancel{x} \times \cancel{y}}$
$= 9xyz$

4. $\dfrac{63x^4yz^2}{x^2yz} = 63 \times \dfrac{\cancel{x} \times \cancel{x} \times x \times x \times \cancel{y} \times \cancel{z} \times z}{\cancel{x} \times \cancel{x} \times \cancel{y} \times \cancel{z}}$
$= 63x^2z$

5. $\dfrac{42m^4n^4p^3}{7m^2np^2} = \dfrac{42}{7} \times (m^4m^{-2}\ n^4n^{-1}\ p^3p^{-2})$
$= 6(m^{4-2}\ n^{4-1}\ p^{3-2})$
$= 6m^2n^3p$

6. $\dfrac{8x^5y^3z}{2x^3y} = \dfrac{8}{2} \times x^{5-3}y^{3-1}z = 4x^2y^2z$

7. $\dfrac{10x^4y^3z^2}{-5x^2y^2z} = \dfrac{10}{-5} \times x^{4-2}y^{3-2}z^{2-1} = -2x^2yz$

8. $\dfrac{72ab^2c^2}{8bc^2} = \dfrac{72}{8} \times \dfrac{a \times b \times \cancel{b} \times \cancel{c} \times \cancel{c}}{\cancel{b} \times \cancel{c} \times \cancel{c}} = 9ab$

9. $\dfrac{21x^2y^3z^5}{7xyz} = \dfrac{21}{7} \times x^{2-1}y^{3-1}z^{5-1} = 3xy^2z^4$

10. $\dfrac{12x^2y^3z}{4x^2yz} = \dfrac{12}{4}\ \dfrac{\cancel{x} \times \cancel{x} \times \cancel{y} \times y \times y \times \cancel{z}}{\cancel{x} \times \cancel{x} \times \cancel{y} \times \cancel{z}} = 3y^2$

11. $\dfrac{32m^3n^3p^2}{8m^2n^2p} = \dfrac{32}{8} \times mnp = 4mnp$

12. $\dfrac{25x^3y^2z^5}{5xyz^3} = \dfrac{25}{5} \times x^{3-1}y^{2-1}z^{5-3} = 5x^2yz^2$

EXERCISE 7.7

1. $\dfrac{14a^4 - 7a^3 + 14a^2}{7a^2} = \dfrac{14a^4}{7a^2} - \dfrac{7a^3}{7a^2} + \dfrac{14a^2}{7a^2}$
$= \dfrac{^{2}\cancel{14} \times \cancel{a} \times \cancel{a} \times a \times a}{\cancel{7} \times \cancel{a} \times \cancel{a}} - \dfrac{^{1}\cancel{7} \times \cancel{a} \times \cancel{a} \times a}{\cancel{7} \times \cancel{a} \times \cancel{a}} + \dfrac{^{2}\cancel{14} \times \cancel{a} \times \cancel{a}}{\cancel{7} \times \cancel{a} \times \cancel{a}}$
$= 2a^2 - a + 2$

2. $\dfrac{24a^3b^3 + 16a^2b - 12ab^2}{4ab} = \dfrac{24a^3b^3}{4ab} + \dfrac{16a^2b}{4ab} - \dfrac{12ab^2}{4ab}$
$= 6a^2b^2 + 4a - 3b$

3. $\dfrac{3x^2y^2z^2 - 15x^2yz^2 + 6x^2y^3z}{3x^2yz}$
$= \dfrac{3x^2y^2z^2}{3x^2yz} - \dfrac{15x^2yz^2}{3x^2yz} + \dfrac{6x^2y^3z}{3x^2yz}$
$= yz - 5z + 2y^2$

4. $\dfrac{ab^4c^3 - 3a^3b^2c - 6ab}{3ab} = \dfrac{ab^4c^3}{3ab} - \dfrac{3a^3b^2c}{3ab} - \dfrac{6ab}{3ab}$

$= \dfrac{b^3c^3}{3} - a^2bc - 2$

5. $\dfrac{-x^4 + 2x^3 + 4x^2 + 6x}{2x} = \dfrac{-x^4}{2x} + \dfrac{2x^3}{2x} + \dfrac{4x^2}{2x} + \dfrac{6x}{2x}$

$= -\dfrac{x^3}{2} + x^2 + 2x + 3$

6. $\dfrac{\sqrt{2}a^3 + 3\sqrt{2}a^2 + 6a}{2a} = \dfrac{\sqrt{2}a^3}{\sqrt{2}\sqrt{2}a} + \dfrac{3\sqrt{2}a^2}{\sqrt{2}\sqrt{2}a} + \dfrac{6a}{2a}$

$= \dfrac{a^2}{\sqrt{2}} + \dfrac{3a}{\sqrt{2}} + 3$

7. $\dfrac{x^3 - 3x^2 + \frac{3}{2}x}{3x} = \dfrac{x^3}{3x} - \dfrac{3x^2}{3x} + \dfrac{\frac{3}{2}x}{3x}$

$= \dfrac{x^2}{3} - x + \dfrac{1}{2}$

8. $\dfrac{8x^3 + 6x^2 + 4x}{4x} = \dfrac{8x^3}{4x} + \dfrac{6x^2}{4x} + \dfrac{4x}{4x}$

$= 2x^2 + \dfrac{3}{2}x + 1$

9. $\dfrac{3y^5 - 6y^4 + 6y^3 + 12y^2}{3y^2} = \dfrac{3y^5}{3y^2} - \dfrac{6y^4}{3y^2} + \dfrac{6y^3}{3y^2} + \dfrac{12y^2}{3y^2}$

$= y^3 - 2y^2 + 2y + 4$

10. $\dfrac{6x^3 + 4x^2}{x^2} = \dfrac{6x^3}{x^2} + \dfrac{4x^2}{x^2}$

$= 6x + 4$

EXERCISE 7.8

1. (*i*) $x - 3 \overline{)\ x^2 - 5x + 6\ (}$

$x - 2$

$-x^2 + 3x$

$-2x + 6$

$-2x + 6$

0

$\therefore (x^2 - 5x + 6) \div (x - 3) = x - 2$

(*ii*) $x - 1 \overline{)\ x^3 - 1\ (}$

$x^2 + x + 1$

$-x^3 + x^2$

$x^2 - 1$

$-x^2 + x$

$x - 1$

$x - 1$

0

$\therefore (x^3 - 1) \div (x - 1) = x^2 + x + 1$

(*iii*) $3x + 4 \overline{)\ 15x^2 + 41x + 28\ (}$

$5x + 7$

$-15x^2 - 20x$

$21x + 28$

$-21x - 28$

0

$\therefore (15x^2 + 41x + 28) \div (3x + 4) = 5x + 7$

(*iv*) $2x + 3 \overline{)\ 2x^3 - 3x^2 + 5x + 21\ (}$

$x^2 - 3x + 7$

$-2x^3 - 3x^2$

$-6x^2 + 5x + 21$

$+6x^2 + 9x$

$14x + 21$

$14x + 21$

0

$\therefore (2x^3 - 3x^2 + 5x + 21) \div (2x + 3) = x^2 - 3x + 7$

(*v*) $2a + 3b \overline{)\ 8a^3 + 27b^3\ (}$

$4a^2 - 6ab + 9b^2$

$-8a^3 + 12a^2b$

$-12a^2b + 27b^3$

$+12a^2b - 18ab^2$

$18ab^2 + 27b^3$

$18ab^2 + 27b^3$

0

$\therefore (8a^3 + 27b^3) \div (2a + 3b) = 4a^2 - 6ab + 9b^2$

(*vi*) $2x + 3 \overline{)\ 10x^2 + 11x - 6\ (}$

$5x - 2$

$-10x^2 + 15x$

$-4x - 6$

$-4x - 6$

0

$\therefore (10x^2 + 11x - 6) \div (2x + 3) = 5x - 2$

(*vii*) $x^2 + 7x - 8 \overline{)\ 2x^4 + 9x^3 - 47x^2 + 68x - 32\ (}$

$2x^2 - 5x + 4$

$-2x^4 - 14x^3 + 16x^2$

$-5x^3 - 31x^2 + 68x - 32$

$+5x^3 + 35x^2 - 40x$

$4x^2 + 28x - 32$

$4x^2 + 28x - 32$

0

$\therefore (2x^4 + 9x^3 - 47x^2 + 68x - 32) \div (x^2 + 7x - 8) = 2x^2 - 5x + 4$

(viii) $2a^2 + 3b^2 \overline{)\ 6a^4 + 11a^2b^2 + 3b^4\ (}\ 3a^2 + b^2$

$-6a^4 \pm 9a^2b^2$

$2a^2b^2 + 3b^4$

$-2a^2b^2 \mp 3b^4$

0

$\therefore\ (6a^4 + 11a^2b^2 + 3b^4) \div (2a^2 + 3b^2) = 3a^2 + b^2$

(ix) $2x + 3 \overline{)\ 2x^3 - 3x^2 + 5x + 21\ (}\ x^2 - 3x + 7$

$-2x^3 \mp 3x^2$

$-6x^2 + 5x + 21$

$+6x^2 \mp 9x$

$14x + 21$

$-14x \pm 21$

0

$\therefore\ (2x^3 - 3x^2 + 5x + 21) \div (2x + 3) = x^2 - 3x + 7$

(x) $3x^2 - 5x + 4 \overline{)\ 6x^4 - x^3 - 28x^2 + 47x - 28\ (}\ 2x^2 + 3x - 7$

$-6x^4 + 10x^3 - 8x^2$

$9x^3 - 36x^2 + 47x - 28$

$-9x^3 + 15x^2 - 12x$

$-21x^2 + 35x - 28$

$+21x^2 - 35x + 28$

0

$\therefore\ (6x^4 - x^3 - 28x^2 + 47x - 28) \div (3x^2 - 5x + 4) = 2x^2 + 3x - 7$

2. (i) $x^2 - 2x + 2 \overline{)\ x^4 + 8x + 15\ (}\ x^2 + 2x + 2$

$-x^4 \quad + 2x^3 - 2x^2$

$2x^3 - 2x^2 + 8x + 15$

$-2x^3 + 4x^2 - 4x$

$2x^2 + 4x + 15$

$-2x^2 + 4x - 4$

$8x + 11$

Quotient: $x^2 + 2x + 2$

Remainder : $8x + 11$

(ii) $x^2 - 2 \overline{)\ x^4 + x^2\ (}\ x^2 + 3$

$-x^4 + 2x^2$

$3x^2$

$-3x^2 + 6$

6

Quotient: $x^2 + 3$

Remainder : 6

(iii) $2a - 3 \overline{)\ 6a^3 - a^2 - 10a - 3\ (}\ 3a^2 + 4a + 1$

$-6a^3 + 9a^2$

$8a^2 - 10a - 3$

$-8a^2 + 12a$

$2a - 3$

$-2a + 3$

0

Quotient: $3a^2 + 4a + 1$

Remainder : 0

(iv) $7x - 4 \overline{)\ 14x^2 + 13x - 15\ (}\ 2x + 3$

$-14x^2 + 8x$

$21x - 15$

$-21x + 12$

-3

Quotient: $2x + 3$

Remainder : -3

3. (i) If $x^3 + 1$ is a factor of $x^5 + x^4 + x^3 + x^2 + x + 1$, then on dividing $x^5 + x^4 + x^3 + x^2 + x + 1$ by $x^3 + 1$ the remainder should be zero. Therefore,

$x^3 + 1 \overline{)\ x^5 + x^4 + x^3 + x^2 + x + 1\ (}\ x^2 + x + 1$

$-x^5 \quad + x^2$

$x^4 + x^3 \quad + x + 1$

$-x^4 \quad + x$

$x^3 \quad + 1$

$-x^3 \quad + 1$

0

Hence, $(x^3 + 1)$ is a factor of $x^5 + x^4 + x^3 + x^2 + x + 1$ and $(x^2 + x + 1)$ is the another factor.

(*ii*) If $(4x - 3)$ is a factor of $8x^2 + 14x - 15$, then on dividing $(8x^2 + 14x - 15)$ by $4x - 3$, the remainder should be zero.

Therefore,

$$
\begin{array}{r|l}
4x-3 & 2x+5 \\
\hline
 & 8x^2+14x-15 \\
 & 8x^2-6x \\
 & \hline
 & 20x-15 \\
 & 20x-15 \\
 & \hline
 & 0
\end{array}
$$

Hence, $(4x - 3)$ is a factor of $8x^2 + 14x - 15$ and $(2x + 5)$ is the another factor.

4. (*i*) If $(3x + 4)$ is a factor of $15x^2 + 41x + 28$, then on dividing $15x^2 + 41x + 28$ by $(3x + 4)$, the remainder should be zero. Therefore,

$$
\begin{array}{r|l}
3x+4 & 5x+7 \\
\hline
 & 15x^2+41x+28 \\
 & 15x^2+20x \\
 & \hline
 & 21x+28 \\
 & 21x+28 \\
 & \hline
 & 0
\end{array}
$$

Hence, $(3x + 4)$ is a factor of $15x^2 + 41x + 28$.

(*ii*) If $(x + y)$ is a factor of $x^4 - y^4$, then on dividing $x^4 - y^4$ by $(x + y)$, the remainder should be zero. Therefore,

$$
\begin{array}{r|l}
x+y & x^3-x^2y+xy^2-y^3 \\
\hline
 & x^4-y^4 \\
 & x^4+x^3y \\
 & \hline
 & -x^3y-y^4 \\
 & -x^3y-x^2y^2 \\
 & \hline
 & x^2y^2-y^4 \\
 & x^2y^2+xy^3 \\
 & \hline
 & -xy^3-y^4 \\
 & -xy^3-y^4 \\
 & \hline
 & 0
\end{array}
$$

Hence, $(x + y)$ is a factor of $x^4 - y^4$.

EXERCISE 7.9

1. $\dfrac{15x^2 + x - 6}{(3x + 2)} = \dfrac{15x^2 + (10 - 9)x - 6}{(3x + 2)}$

$= \dfrac{15x^2 + 10x - 9x - 6}{(3x + 2)}$

$= \dfrac{5x(3x + 2) - 3(3x + 2)}{(3x + 2)}$

$= \dfrac{(3x + 2)(5x - 3)}{(3x + 2)}$

($\because$ Cancelling $(3x + 2)$ from numerator and denominator)

$= (5x - 3)$

2. $\dfrac{4x^2 - 8x + 3}{(2x - 1)} = \dfrac{4x^2 - (6 + 2)x + 3}{(2x - 1)}$

$= \dfrac{4x^2 - 6x - 2x + 3}{(2x - 1)}$

$= \dfrac{2x(2x - 3) - 1(2x - 3)}{(2x - 1)}$

$= \dfrac{(2x - 3)(2x - 1)}{(2x - 1)} = (2x - 3)$

3. $\dfrac{x^2 + 12x + 20}{(x + 10)} = \dfrac{x^2 + (10 + 2)x + 20}{(x + 10)}$

$= \dfrac{x^2 + 10x + 2x + 20}{(x + 10)}$

$= \dfrac{x(x + 10) + 2(x + 10)}{(x + 10)}$

$= \dfrac{(x + 10)(x + 2)}{(x + 10)} = (x + 2)$

4. $\dfrac{x^2 - 14x - 51}{(x + 3)} = \dfrac{x^2 - (17 - 3)x - 51}{(x + 3)}$

$= \dfrac{x^2 - 17x + 3x - 51}{(x + 3)}$

$= \dfrac{x(x - 17) + 3(x - 17)}{(x + 3)}$

$= \dfrac{(x - 17)(x + 3)}{(x + 3)} = (x - 17)$

5. $\dfrac{3m^2 + 22m + 35}{(3m + 7)} = \dfrac{3m^2 + (15 + 7)m + 35}{(3m + 7)}$

$= \dfrac{3m^2 + 15m + 7m + 35}{(3m + 7)}$

$= \dfrac{3m(m + 5) + 7(m + 5)}{(3m + 7)}$

$= \dfrac{(m + 5)(3m + 7)}{(3m + 7)} = (m + 5)$

6. $\frac{m^2+8m+12}{(m+2)} = \frac{m^2+(6+2)m+12}{(m+2)}$

$= \frac{m^2+6m+2m+12}{(m+2)}$

$= \frac{m(m+6)+2(m+6)}{(m+2)}$

$= \frac{(m+6)(m+2)}{(m+2)} = (m+6)$

7. $\frac{x^4+3x^2}{3x^3+9x} = \frac{x^2(x^2+3)}{3x(x^2+3)}$

$= \frac{x \times x \times (x^2+3)}{3 \times x \times (x^2+3)} = \frac{x}{3}$

8. $\frac{8x^2-18x+9}{(2x-3)} = \frac{8x^2-(12+6)x+9}{(2x-3)}$

$= \frac{8x^2-12x-6x+9}{(2x-3)}$

$= \frac{4x(2x-3)-3(2x-3)}{(2x-3)}$

$= \frac{(2x-3)(4x-3)}{(2x-3)} = (4x-3)$

9. $\frac{6x^2+11x-10}{(2x+5)} = \frac{6x^2+(15-4)x-10}{(2x+5)}$

$= \frac{6x^2+15x-4x-10}{(2x+5)}$

$= \frac{3x(2x+5)-2(2x+5)}{(2x+5)}$

$= \frac{(2x+5)(3x-2)}{(2x+5)} = (3x-2)$

10. $\frac{2x^2+x-45}{(x+5)} = \frac{2x^2+(10-9)x-45}{(x+5)}$

$= \frac{2x^2+10x-9x-45}{(x+5)}$

$= \frac{2x(x+5)-9(x+5)}{(x+5)}$

$= \frac{(x+5)(2x-9)}{(x+5)} = (2x-9)$

11. $\frac{x^2-17x+16}{(x-16)} = \frac{x^2-(16+1)x+16}{(x-16)}$

$= \frac{x^2-16x-x+16}{(x-16)}$

$= \frac{x(x-16)-1(x-16)}{(x-16)}$

$= \frac{(x-16)(x-1)}{(x-16)} = (x-1)$

12. $\frac{9x^2-12x+4}{(3x-2)} = \frac{9x^2-(6+6)x+4}{(3x-2)}$

$= \frac{9x^2-6x-6x+4}{(3x-2)}$

$= \frac{3x(3x-2)-2(3x-2)}{(3x-2)}$

$= \frac{(3x-2)(3x-2)}{(3x-2)} = (3x-2)$

13. $\frac{x^6-9x^2}{(x^3-3x)} = \frac{x^2(x^4-9)}{x(x^2-3)}$

$= \frac{x^2\left[(x^2)^2-(3)^2\right]}{x(x^2-3)}$

$= \frac{x^2(x^2+3)(x^2-3)}{x(x^2-3)}$

$[\because \quad a^2-b^2 = (a+b)(a-b)]$

$= x(x^2+3)$

14. $\frac{6x^3+18x^2+12x}{2x^2+4x} = \frac{6x(x^2+3x+2)}{2x(x+2)}$

$= \frac{3(x^2+2x+x+2)}{(x+2)}$

$= \frac{3\left[x(x+2)+(x+2)\right]}{(x+2)}$

$= \frac{3(x+2)(x+1)}{(x+2)}$

$= 3(x+1)$

$= (3x+3)$

MULTIPLE CHOICE QUESTIONS

1. Factors of $9x^2y = 3 \times 3 \times x \times x \times y$

Factors of $27xy^2 = 3 \times 3 \times 3 \times x \times y \times y$

Factors of $45x^2y^2 = 3 \times 3 \times 5 \times x \times x \times y \times y$

The greatest common factor of $9x^2y$, $27xy^2$ and $45x^2y^2$ $= 3 \times 3 \times x \times y = 9xy$

Hence option (*d*) is correct.

2. $9x - 18x^2 = 9x\ (1 - 2x)$

Hence, option (*b*) is correct.

3.

$$
\begin{array}{r}
x^2 + 6x - 1 \\
x - 1 \overline{)\ x^3 + 5x^2 - 7x + 4\ (} \\
\underline{-\ x^3 \mp x^2} \\
6x^2 - 7x + 4 \\
\underline{-\ 6x^2 \mp 6x} \\
-x + 4 \\
\underline{+\ -\ x \pm 1} \\
3
\end{array}
$$

Remainder = 3

Hence, option (*a*) is correct.

4. $x^2 + xy - 5x - 5y = (x^2 + xy) - (5x + 5y)$

$= x(x + y) - 5(x + y)$

$= (x + y)\ (x - 5)$

Hence, option (*c*) is correct.

5. $\dfrac{-15a^2bc^2}{5ab} = \dfrac{-15}{5} \times \dfrac{\cancel{a} \times a \times \cancel{b} \times c \times c}{\cancel{a} \times \cancel{b}}$

$= -3ac^2$

Hence, option (*b*) is correct.

6.

$$
\begin{array}{r}
x^3 + 5 \\
x - 1 \overline{)\ x^4 - x^3 + 5x\ (} \\
\underline{-\ x^4 \mp x^3} \\
5x \\
\underline{-\ \ +\ \ 5x - 5} \\
5
\end{array}
$$

Remainder = 5

Hence, option (*c*) is correct.

7. $\dfrac{27a^3b^2}{-9ab} = \dfrac{27}{-9} \times \dfrac{\cancel{a} \times a \times a \times b \times \cancel{b}}{\cancel{a} \times \cancel{b}}$

$= -3a^2b$

Hence, option (*a*) is correct.

8.

$$
\begin{array}{r}
x + 2 \\
x + 2 \overline{)\ x^2 + 4x + 4\ (} \\
\underline{-\ \ -\ \ x^2 + 2x} \\
2x + 4 \\
\underline{-\ \ -\ \ 2x + 4} \\
0
\end{array}
$$

Quotient $= (x + 2)$

Hence, option (*d*) is correct.

9. $\dfrac{9x^2 - 1}{3x - 1} = \dfrac{(3x)^2 - 1^2}{3x - 1} = \dfrac{(3x + 1)(3x - 1)}{(3x - 1)} = 3x + 1$

Quotient = $3x + 1$

Hence, option (*b*) is correct.

10. $x^4 - x = x(x^3 - 1) = x(-1 + x^3)$

Hence, option (*b*) is correct.

11. $(2x - 8x^3) = 2x(1 - 4x^2)$

$= 2x[1^2 - (2x)^2]$

$= 2x(1 + 2x)\ (1 - 2x)$

Hence, option (*c*) is correct.

12. $xy - x - y + 1 = (xy - x) - (y - 1)$

$= x(y - 1) - (y - 1)$

$= (y - 1)\ (x - 1)$

$= (x - 1)\ (y - 1)$

Hence, option (*c*) is correct.

MENTAL MATHS CORNER

1. $(x + 3)$ is a factor of $x^3 + 2x^2 - 3x + 6$. **(False)**

$$
\begin{array}{r}
\because \quad x^2 - x \\
x + 3 \overline{)\ x^3 + 2x^2 - 3x + 6\ (} \\
\underline{-\ \ -\ \ x^3 + 3x^2} \\
-x^2 - 3x + 6 \\
\underline{-\ \ +\ \ x^2 \mp 3x} \\
6
\end{array}
$$

Remainder = $6 \neq 0$. So it is not a factor.

2. $(x - 2)$ is a factor of $x^3 - 8$. **(True)**

$$
\begin{array}{r}
\because \quad x^2 + 2x + 4 \\
x - 2 \overline{)\ x^3 - 8\ (} \\
\underline{-\ \ \ x^3 \ \ -2x^2} \\
2x^2 - 8 \\
\underline{-\ \ \ 2x^2 \ \ -4x} \\
4x - 8 \\
\underline{-\ \ +\ \ 4x - 8} \\
0
\end{array}
$$

Remainder =0. So it is a factor.

3. The factor of $xy - ab + bx - ay$ are $(y - b)\ (x + a)$.

(False)

$\because \quad xy - ab + bx - ay = (xy + bx) - (ab + ay)$

$= x(y + b) - a(b + y)$

$= x(y + b) - a(y + b)$

$= (y + b)\ (x - a)$

4. $(x + 1)$ is a factor of $2x^2 + 5x + 4$. **(False)**

$$\begin{array}{r} 2x+3 \\ x+1\overline{)\ 2x^2+5x+4\ (} \\ \underline{-\ \ 2x^2+2x\ \ \ \ } \\ 3x+4 \\ \underline{-\ \ 3x+3\ \ } \\ 1 \end{array}$$

Remainder = $1 \neq 0$. So it is not a factor

5. $2x^2 + 3x + 1$ is divided by $(x + 1)$, then the remainder is zero. **(True)**

$$\begin{array}{r} 2x+1 \\ x+1\overline{)\ 2x^2+3x+1\ (} \\ \underline{-\ \ 2x^2+2x\ \ \ \ } \\ x+1 \\ \underline{-\ \ x+1\ \ } \\ 0 \end{array}$$

Remainder = 0.

6. $\dfrac{a^4 - b^4}{a - b} = (a + b)\ (a^2 + b^2)$ **(True)**

$$\therefore \quad \frac{a^4 - b^4}{a - b} = \frac{(a^2)^2 - (b^2)^2}{a - b} = \frac{(a^2 + b^2)\,(a^2 - b^2)}{(a - b)}$$

$$= \frac{(a^2 + b^2)\,(a + b)\,(a - b)}{(a - b)} = (a + b)(a^2 + b^2)$$

REVIEW EXERCISE

1. $(i)\ 4a^2 - 8a + 3 = 4a^2 - (6 + 2)a + 3$

$= 4a^2 - 6a - 2a + 3$

$= 2a\ (2a - 3) - 1(2a - 3)$

$= (2a - 3)\ (2a - 1)$

$\therefore \quad 4a^2 - 8a + 3 = (2a - 3)(2a - 1)$

$(ii)\ 6x^2 + 35xy - 6y^2 = 6x^2 + (36 - 1)xy - 6y^2$

$= 6x^2 + 36xy - xy - 6y^2$

$= 6x\ (x + 6y) - y(x + 6y)$

$= (x + 6y)\ (6x - y)$

$\therefore \quad 6x^2 + 35xy - 6y^2 = (x + 6y)\ (6x - y)$

$(iii)\ 16 - a^2 - 2ab - b^2 = 16 - (a^2 + 2ab + b^2)$

$= 16 - (a + b)^2$

$= 4^2 - (a + b)^2$

$= [4 + (a + b)]\ [4 - (a + b)]$

$= (4 + a + b)\ (4 - a - b)$

$(iv)\ 2 - 32x^2 = 2(1 - 16x^2)$

$= 2[1^2 - (4x)^2]$

$= 2(1 + 4x)\ (1 - 4x)$

$\therefore \quad 2 - 32x^2 = 2(1 + 4x)\ (1 - 4x)$

2.

$$\begin{array}{r} 2x^3+2x^2+x+1 \\ x+3\overline{)\ 2x^4+8x^3+7x^2+4x+3\ (} \\ \underline{-\ \ 2x^4+6x^3\ \ \ \ \ \ \ \ } \\ 2x^3+7x^2+4x+3 \\ \underline{-\ \ 2x^3+6x^2\ \ \ \ \ } \\ x^2+4x+3 \\ \underline{-\ \ x^2+3x\ \ \ } \\ x+3 \\ \underline{-\ \ x+3\ \ } \\ 0 \end{array}$$

Quotient $= 2x^3 + 2x^2 + x + 1$

Remainder = 0

3. If $(x + 2)$ is a factor of $4x^4 + 2x^3 - 3x^2 + 8x + 109$, then on dividing $4x^4 + 2x^3 - 3x^2 + 8x + 109$ by $(x + 2)$, the remainder should be zero. Therefore,

$$\begin{array}{r} 4x^3-6x^2+9x-10 \\ x+2\overline{)\ 4x^4+2x^3-3x^2+8x+109\ (} \\ \underline{-\ \ 4x^4+8x^3\ \ \ \ \ \ \ \ \ \ } \\ -6x^3-3x^2+8x+109 \\ \underline{-\ 6x^3-12x^2\ \ \ \ \ \ } \\ 9x^2+8x+109 \\ \underline{-\ \ 9x^2+18x\ \ \ } \\ -10x+109 \\ \underline{-\ 10x-20\ \ } \\ 129 \end{array}$$

$\therefore \quad$ Remainder $= 129 \neq 0$

Hence, $(x + 2)$ is not a factor of $4x^4 + 2x^3 - 3x^2 + 8x + 109$.

4. L.H.S. $= (x - y)(x + y) + (y - z)(y + z) + (z - x)(z + x)$

$= (x^2 - y^2) + (y^2 - z^2) + (z^2 - x^2)$ $[\because \quad (a - b)(a + b) = a^2 - b^2]$

$= x^2 - y^2 + y^2 - z^2 + z^2 - x^2$

$= 0$

$=$ R.H.S.

Hence, $(x - y)(x + y) + (y - z)(y + z) + (z - x)(z + x) = 0$

5. $4x = (56)^2 - (52)^2$

$\Rightarrow \quad 4x = (56 - 52)(56 + 52)$

$[\because \quad a^2 - b^2 = (a - b)(a + b)]$

$\Rightarrow \quad 4x = 4 \times 108$

$\Rightarrow \quad x = \frac{4 \times 108}{4} = 108$

$\Rightarrow \quad \boxed{x = 108}$

6. L.H.S. $= \frac{(x^2 + 2xy + y^2) - a^2 + 2ab - b^2}{(x + y - a + b)}$

$= \frac{(x^2 + 2xy + y^2) - (a^2 - 2ab + b^2)}{(x + y - a + b)}$

$= \frac{(x + y)^2 - (a - b)^2}{(x + y - a + b)}$

$= \frac{[x + y + (a - b)][x + y - (a - b)]}{(x + y - a + b)}$

$[\because \quad a^2 - b^2 = (a + b)(a - b)]$

$= \frac{(x + y + a - b)(x + y - a + b)}{(x + y - a + b)}$

$= (x + y + a - b)$

= R.H.S.

Hence, $\frac{x^2 + 2xy + y^2 - a^2 + 2ab - b^2}{(x + y - a + b)} = (x + y + a - b)$

7. $(2a + 3b)^2 - 5(2a + 3b) - 14$

Let $(2a + 3b) = x$

Then, $x^2 - 5x - 14 = x^2 - (7 - 2)x - 14$

$= x^2 - 7x + 2x - 14$

$= x(x - 7) + 2(x - 7)$

$= (x - 7)(x + 2)$

$\therefore \quad (2a + 3b)^2 - 5(2a + 3b) - 14$

$= (2a + 3b - 7)(2a + 3b + 2)$

8. $(x^2 - 5x)^2 - 36$

Let $x^2 - 5x = m$, then

$m^2 - 36 = (m - 6)(m + 6)$

$[\because \quad a^2 - b^2 = (a - b)(a + b)]$

$= (x^2 - 5x - 6)(x^2 - 5x + 6)$

$= [x^2 - (6 - 1)x - 6][x^2 - (2 + 3)x + 6]$

$= [x^2 - 6x + x - 6][x^2 - 2x - 3x + 6]$

$= [x(x - 6) + 1(x - 6)][x(x - 2) - 3(x - 2)]$

$= [(x - 6)(x + 1)][(x - 2)(x - 3)]$

$= (x + 1)(x - 2)(x - 3)(x - 6)$

9.

$$3x - 6 \overline{)\ 15x^3 - 20x^2 + 13x - 12\ (\ 5x^2 + \frac{10}{3}x + 11}$$

$$\underline{-\ 15x^3 \mp 30x^2}$$

$$10x^2 + 13x - 12$$

$$\underline{-\ 10x^2 \mp 20x}$$

$$33x - 12$$

$$\underline{-\ \ +\ \ 33x - 66}$$

$$54$$

Quotient = $5x^2 + \frac{10}{3}x + 11$, Remainder = 54

To verify:

Divisor × Quotient + Remainder

$= (3x - 6) \times \left(5x^2 + \frac{10}{3}x + 11\right) + 54$

$= 3x \times 5x^2 + 3x \times \frac{10}{3}x + 3x \times 11 - 6 \times 5x^2$

$- 6 \times \frac{10}{3}x - 6 \times 11 + 54$

$= 15x^3 + 10x^2 + 33x - 30x^2 - 20x - 66 +$

54

$= 15x^3 - 20x^2 + 13x - 12$

= Dividend

Hence, Dividend = Divisor × Quotient + Remainder

10. $\frac{p^4 - q^4}{(p - q)} = \frac{(p^2 + q^2)(p^2 - q^2)}{(p - q)}$

$[\because \quad a^2 - b^2 = (a + b)(a - b)]$

$= \frac{(p^2 + q^2)(p + q)(p - q)}{(p - q)}$

$\therefore \quad \frac{p^4 - q^4}{(p - q)} = (p + q)(p^2 + q^2)$

HOTS QUESTIONS

1. (*i*) $a^4 - (a - b)^4 = [(a^2)^2 - \{(a - b)^2\}^2]$

$= [a^2 - (a - b)^2]\ [a^2 + (a - b)^2]$

$[\because \quad a^2 - b^2 = (a - b)(a + b)]$

$= [a - (a - b)][a + (a - b)][a^2 + (a - b)^2]$

$= (a - a + b)(a + a - b)(a^2 + a^2 - 2ab + b^2)$

$= (2a^2 - 2ab + b^2)(2a - b)b$

(*ii*) $a(a + b - c) - bc = a^2 + ab - ac - bc$

$= (a^2 + ab) - (ac + bc)$

$= a(a + b) - c(a + b)$

$= (a + b)(a - c)$

(*iii*) $16(2a - 1)^2 - 25b^2 = [4(2a - 1)]^2 - (5b)^2$

$= [4(2a - 1) + 5b][4(2a - 1) - 5b]$

$= (8a - 4 + 5b)(8a - 4 - 5b)$

$= (8a + 5b - 4)(8a - 5b - 4)$

2. $\dfrac{(3x^2+24x+36)(x+4)}{3(x^2+6x+8)} = \dfrac{[3x^2+(18+6)x+36](x+4)}{3[x^2+(4+2)x+8]}$

$$= \frac{[(3x^2+18x+6x+36)](x+4)}{3(x^2+4x+2x+8)}$$

$$= \frac{[3x(x+6)+6(x+6)](x+4)}{3\left[x\left(x+4\right)+2\left(x+4\right)\right]}$$

$$= \frac{(x+6)(3x+6)(x+4)}{3(x+2)(x+4)}$$

$$= \frac{3(x+6)(x+2)(x+4)}{3(x+2)(x+4)}$$

$$= (x+6)$$

3. $a^3x + a^2(x-y) - a(y+z) - z$

$= a^3x + a^2x - a^2y - ay - az - z$

$= (a^3x + a^2x) - (a^2y + ay) - (az + z)$

$= a^2x(a+1) - ay(a+1) - z(a+1)$

$= (a+1)(a^2x - ay - z)$

Puzzle

Total distance between the Haretown and Tortoiseville = 54 km.

Speed of hare = 7 km/hr

Let the distance travelled by the hare before meeting the tortoise from Haretown to Tortoiseville be x km.

$\therefore$ Tortoise will travel a distance of $(54 - x)$ km from Tortoiseville to Haretown.

Speed of tortoise = 2 km/hr

Now,

The time taken by the hare to cover a distance of x km at 7 km/hr = $\dfrac{x}{7}$ hours

The time taken by the tortoise to cover a distance of $(54 - x)$ km at 2 km/hr = $\dfrac{54-x}{2}$ hours

Since, both set out at the same time. Therefore,

$$\frac{x}{7} = \frac{54-x}{2}$$

$\Rightarrow \quad 2x = 7(54 - x)$

$\Rightarrow \quad 2x = 378 - 7x$

$\Rightarrow \quad 2x + 7x = 378 \quad \Rightarrow \quad 9x = 378$

$\Rightarrow \quad x = \dfrac{378}{9} = 42$

Hence, hare will travel 42 km before meeting the tortoise.